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The β-term for $D^* \to D\gamma$ within a heavy-light chiral quark model

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Abstract. We present a calculation of the β-term for $D^* \to D\gamma$ within a quark model which exhibits both chiral symmetry and heavy quark symmetry. Soft gluon effects in terms of a model dependent gluon condensate are included. Also, calculations of $1/m_c$ corrections are performed. We find that the value of β is rather sensitive to the constituent quark mass compared to other quantities calculated within the same model. Also, to obtain a value close to the experimental value, one has to choose a constituent light quark mass larger than for other quantities studied in previous papers. For a light quark mass in the range 250 to 300 MeV and a quark condensate in the range -(250 - 270 MeV)^3 we find the value (2.5 $\pm$ 0.6) GeV^{-1}. This value is in agreement with the value of β extracted from experiment (2.7 $\pm$ 0.2) GeV^{-1}.

1 Introduction

Strong and electromagnetic interactions involving heavy and light mesons and photons may be described by heavy-light chiral Lagrangians [1]. Such Lagrangians are determined by chiral symmetry in the light sector in addition to heavy quark symmetry. However, the coupling constants of the terms in the Lagrangian are in general unknown. In this paper we focus on the so called β-term for $D^* \to D\gamma$, which is studied in [2,4,3]. This term in the Lagrangian is also relevant for calculation of processes like $D^0 \to 2\gamma$ [5], and $D^0 \to e^+e^-\gamma$ [6].

Within heavy-light chiral perturbation theory alone, the value of β is not determined. In this paper we will calculate the β-term to order $1/m_c$ within a Heavy-Light Chiral Quark Model (HLχQM) developed recently [7]. This model belongs to a class of quark loop models [8,9,10] where the quarks couple directly to the mesons at the scale of chiral symmetry breaking Λ_χ of order 1 GeV. In contrast to most other versions of such models, our version also incorporates soft gluon effects in terms of the gluon condensate with lowest dimension [7,11,16,13,14, 15,12]. This gluon condensate, together with the (light) quark condensate, the constituent light quark mass and the coupling of quarks to the heavy meson, are model dependent parameters which can to a certain extent be linked to physical parameters.

At quark level the Lagrangian of our model [7] includes the Lagrangian for Heavy Quark Effective Field Theory (HQEFT) [17]. This means that $1/m_c$ corrections can be calculated within the model. It has been problematic to pin down a precise value for β, and chiral loop contributions to $D^* \to D\gamma$ are sizeable [2,4,3]. This may indicate that the chiral expansion will be problematic for β.

2 The value of β extracted from experiment

The width of the D^{*+} meson has been measured very accurately [18]:

$$\Gamma(D^{*+}) = (96 \pm 26) \text{ keV} \tag{1}$$

from this value of the width it is possible to extract a value for the $D^*D\pi$ coupling[18]

$$g_{\mathcal{A}}{}^{D^*D\pi} = 0.59 \pm 0.08 \, . \tag{2}$$

From (1) we find the following radiative decay rate:

$$\Gamma(D^{*+} \to D^+\gamma) = (1.54 \pm 0.56) \text{ keV} \, , \tag{3}$$

where we have used the branching ratio $Br(D^{*+} \to D^+\gamma)$ = $(1.6 \pm 0.4)\%$ [19]. To one-loop in the chiral expansion and to order $1/m_c$ we have [2,3] :

$$\Gamma(D^{*+} \to D^+\gamma) = \frac{\omega^3\alpha}{3M_{D^*}}\sqrt{M_{D^*}M_D}\left[-\frac{1}{3}\beta_d + \frac{2}{3}\frac{1}{m_c} + \frac{g_{\mathcal{A}}{}^2}{8\pi}\frac{m_\pi}{f^2}\right]^2 \, , \tag{4}$$

where f is the bare pion decay constant $f = 86$ MeV[3], and ω is the photon energy $\omega = (M_{D^*}^2 - M_D^2)/(2M_{D^*})$. The term $1/m_c$ is due to photon emission from the c-quark. The bare coupling $g_{\mathcal{A}}$ is related to $g_{\mathcal{A}}{}^{D^*D\pi}$. In the Heavy-Light Chiral Quark Model [7], described to a certain extent in the next section, it is estimated to be $g_\mathcal{A} = 0.57 \pm 0.05$. This estimate includes chiral corrections to one loop and $1/m_Q$ corrections [7]. This value is very close to (2), in the following we will use the experimental value as the bare coupling. The quantity β_d is the effective, renormalized β including chiral corrections for the case D^+ when the D^* and D mesons contain a d-quark [3]:

$$\beta_d = \beta \left\{ 1 + {g_\mathcal{A}}^2 \left(\frac{9}{2}\varepsilon_\pi + 3\varepsilon_K + \frac{1}{2}\varepsilon_{\eta_8} \right) + 3\left(\varepsilon_\pi + \frac{{g_\mathcal{A}}^2}{3}\left(-\frac{3}{2}\varepsilon_\pi + \varepsilon_K + \frac{1}{6}\varepsilon_{\eta_8} \right) \right) \right\} \quad , \quad (5)$$

where we have used the notation:

$$\varepsilon_X = \frac{m_X^2}{32\pi^2 f^2} \ln \frac{\Lambda_\chi^2}{m_X^2} \quad , \quad X = \pi, K, \eta_8 \; . \quad (6)$$

Equation (5) includes vertex corrections for the β-term and field renormalization. Combining equation (3) and (4), we find two possible solutions for β_d:

$$\beta_d = (0.7 \pm 0.5)\ \mathrm{GeV}^{-1} \quad , \quad \beta_d = (4.1 \pm 0.2)\ \mathrm{GeV}^{-1} \quad (7)$$

where we have used $m_Q = m_c = 1.4$ GeV. These values correspond to the bare values

$$\beta = (0.7 \pm 0.4)\ \mathrm{GeV}^{-1} \quad , \quad \beta = (2.7 \pm 0.2)\ \mathrm{GeV}^{-1} \quad (8)$$

Dropping the chiral corrections in (4) ($\beta_d = \beta$) would give the result:

$$\beta = -(0.1 \pm 0.3)\ \mathrm{GeV}^{-1} \quad , \quad \beta = (3.0 \pm 0.3)\ \mathrm{GeV}^{-1} \quad (9)$$

The values in (8) and (9) are consistent with those obtained in [5]. Both the first values of β in equations (8) and (9) are too low in order to agree with D^{*0} data [5]. Thus, we are left with the second values of (8) and (9).

3 The heavy light chiral quark model

In this section we will give a short description of the Heavy-Light Chiral Quark Model (HLχQM) to be used in this paper. The model is based on Heavy Quark Effective Theory (HQEFT), which is a systematic expansion in $1/m_Q$ [17] (where m_Q is m_c in our case). The heavy quark field $Q(x)$ is replaced with a "reduced" field, $Q_v(x)$, which is related to the full field the in following way:

$$Q_v(x) = P_+ e^{-im_Q v\cdot x} Q(x) \, , \quad (10)$$

where $P_\pm$ are projection operators $P_\pm = (1 \pm \gamma \cdot v)/2$. The reduced field Q_v annihilates heavy quarks. The Lagrangian for heavy quarks is:

$$\mathcal{L}_{\mathrm{HQEFT}} = \overline{Q_v}\, iv\cdot D\, Q_v + \frac{1}{2m_Q}\overline{Q_v} \left(-C_M \frac{g_s}{2}\sigma\cdot G + (iD_\perp)^2_{\mathrm{eff}} \right) Q_v + \mathcal{O}(m_Q^{-2}) \quad (11)$$

where D_μ is the covariant derivative containing the gluon field (eventually also the photon field), and $\sigma \cdot G = \sigma^{\mu\nu} G^a_{\mu\nu} t^a$, where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, $G^a_{\mu\nu}$ is the gluonic field tensors, and t^a are the colour matrices. This chromomagnetic term has a factor C_M which is one at tree level, but slightly modified by perturbative QCD effects below m_Q. It has been calculated to next to leading order (NLO) [20,21]. Furthermore, $(iD_\perp)^2_{\mathrm{eff}} = C_D(iD)^2 - C_K(iv\cdot D)^2$. At tree level, $C_D = C_K = 1$. Here, C_D is not modified by perturbative QCD, while C_K is different from one due to perturbative QCD corrections [21].

The Lagrangian for the HLχQM is

$$\mathcal{L}_{\mathrm{HL}\chi\mathrm{QM}} = \mathcal{L}_{\mathrm{HQEFT}} + \mathcal{L}_{\chi\mathrm{QM}} + \mathcal{L}_{\mathrm{Int}} \; . \quad (12)$$

The first term is given in equation (11) (Note however, that only soft gluons are considered to be included in (12)) . The light quark sector is described by the Chiral Quark Model (χQM) [8], having a standard QCD term and a term describing interactions between quarks and (Goldstone) mesons:

$$\mathcal{L}_{\chi\mathrm{QM}} = \overline{\chi}\left[\gamma^\mu(iD_\mu + \mathcal{V}_\mu + \gamma_5\mathcal{A}_\mu) - m\right]\chi - \overline{\chi}\widetilde{M_q}\chi \; . \quad (13)$$

Here m is the (SU(3)-invariant) constituent light quark mass and χ is the flavour rotated quark fields given by: $\chi_L = \xi^\dagger q_L$ and $\chi_R = \xi q_R$, where $q^T = (u, d, s)$ are the light quark fields. The left- and right-handed projections q_L and q_R transforms after $SU(3)_L$ and $SU(3)_R$ respectively. The quantity ξ is a 3 by 3 matrix containing the (would be) Goldstone octet (π, K, η). In terms of ξ the vector and axial vector fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ in (13) are given by:

$$\mathcal{V}_\mu \equiv \tfrac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger) \, , \qquad \mathcal{A}_\mu \equiv -\tfrac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) \, , \qquad \xi \equiv \exp\left(i\Pi/f\right) \quad (14)$$

where f is the bare pion coupling, and Π is a 3 by 3 matrix which contains the Goldstone bosons π, K, η in the standard way. In (13) the quantity $\widetilde{M_q}$ contains the current quark mass matrix $\mathcal{M}_q$ and and the Goldstone fields through ξ:

$$\widetilde{M_q} \equiv \widetilde{M_q^V} + \widetilde{M_q^A}\gamma_5 \, , \text{ where} \quad (15)$$

$$\widetilde{M_q^V} \equiv \frac{1}{2}(\xi^\dagger\mathcal{M}_q^\dagger\xi^\dagger + \xi\mathcal{M}_q\xi) \quad \text{and} \quad \widetilde{M_q^A} \equiv -\frac{1}{2}(\xi^\dagger\mathcal{M}_q^\dagger\xi^\dagger - \xi\mathcal{M}_q\xi) \; . \quad (16)$$

The interaction between heavy meson fields and heavy quarks are described by the following Lagrangian:

$$\mathcal{L}_{Int} = -G_H \left[\overline{\chi}_a\, \overline{H_a}\, Q_v + \overline{Q_v}\, H_a\, \chi_a \right] \, , \quad (17)$$

where G_H is a coupling constant, and H_a is the heavy meson field containing a spin zero and spin one boson (a is a SU(3) flavour index):

$$H_a \equiv P_+(P^a_\mu\gamma_\mu - iP^a_5\gamma_5) \quad , \quad \overline{H_a} \equiv \gamma^0(H_a)^\dagger\gamma^0 \; . \quad (18)$$

The fields P^a annihilates a heavy meson containing a heavy quark with velocity v.

Integrating out the quarks by using by using (11), (13) and (17), the lowest order chiral Lagrangian up to $\mathcal{O}(m_Q^{-1})$ can be written as [1,7]

$$\mathcal{L} = -Tr\left[\overline{H_a} iv\cdot\mathcal{D}_{ba} H_b\right] - g_{\mathcal{A}}\, Tr\left[\overline{H_a} H_b \gamma_\mu\gamma_5 \mathcal{A}^\mu_{ba}\right] , \tag{19}$$

where $i\mathcal{D}^\mu_{ba} = i\delta_{ba}D^\mu - \mathcal{V}^\mu_{ba}$ and the axial coupling $g_{\mathcal{A}}$ is of order 0.6. Equations (19) and (14) will be used for the chiral loop contributions. (Note that (17) and (19) both contain an additional term [7] . These are, however, irrelevant in the present paper).

To obtain (19) from the HLχQM one encounters divergent loop integrals, which will be quadratic-, linear- and logarithmic divergent. Thus G_H^2 times linear combinations of these divergent integrals have to be put equal to 1 (the normalization) and $g_{\mathcal{A}}$ respectively. For details, see the Appendix. Within our model, the values for the regularized versions of the quadratic-, linear-, and logarithmic divergent integrals are determined by the physical values of $\langle\,\overline{q}q\,\rangle$, $g_{\mathcal{A}}$, and f respectively. The effective coupling G_H describing the interaction between the quarks and heavy mesons can be expressed in terms of m, f, $g_{\mathcal{A}}$, and the mass splitting between the 1^- state and 0^- state. Using the equations (35), (37), (39), and (40) in the Appendix, one finds the following relations between this mass-splitting and the gluon condensate via the chromomagnetic interaction in (11) [7]:

$$\langle \tfrac{\alpha_s}{\pi} G^2\rangle = \tfrac{16 f^2}{\pi\eta}\,\tfrac{\mu_G^2}{\rho} , \qquad G_H^2 = \tfrac{2m}{f^2}\,\rho ,$$
$$\eta \equiv \tfrac{(\pi+2)}{\pi} C_M(\Lambda_\chi) , \tag{20}$$

where

$$\rho \equiv \frac{(1+3g_{\mathcal{A}}) + \frac{\mu_G^2}{\eta\, m^2}}{4(1+\frac{N_c m^2}{8\pi f^2})} \quad , \quad \mu_G^2(H) = \frac{3}{2} m_Q (M_{H^*} - M_H). \tag{21}$$

When calculating the soft gluon effects in terms of the gluon condensate, we follow [22]. The gluon condensate is obtained by the replacement:

$$g_s^2 G^a_{\mu\nu} G^b_{\alpha\beta} \;\to\; \frac{4\pi^2}{(N_c^2-1)}\delta^{ab}\langle\frac{\alpha_s}{\pi}G^2\rangle \frac{1}{12}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) . \tag{22}$$

Soft gluons coupling to a heavy quark is suppressed by $1/m_Q$, since to leading order the vertex is proportional to $v_\mu v_\nu G^{a\mu\nu} = 0$, v_μ being the heavy quark velocity. We observe that the mass-splitting between H and H^* sets the scale of the gluon condensate. Within the pure light sector, the gluon condensate contribution obtained by using (22) has apriori an arbitrary value. One may assume that it has the same value as in QCD sum rules, or it has to be fitted to some effect, in our case the mass splitting between pseudoscalar and vector heavy mesons.

The $1/m_Q$ corrections to the strong Lagrangian have been calculated in [7]. They may formally be put into spin dependent renormalization factors. This means that (19) is still valid with the replacement $H \to H^{\rm r} = H\,(Z_H)^{-\frac{1}{2}}$, where Z_H and the renormalized (effective) coupling $\tilde{g}_{\mathcal{A}}$ are defined as:

$$Z_H^{-1} = 1 + \frac{\varepsilon_1 - 2d_M\varepsilon_2}{m_Q} , \tag{23}$$

$$\tilde{g}_{\mathcal{A}} = g_{\mathcal{A}}\left(1 - \frac{1}{m_Q}(\varepsilon_1 - 2d_{\mathcal{A}}\varepsilon_2)\right) - \frac{1}{m_Q}(g_1 - d_{\mathcal{A}} g_2) , \tag{24}$$

where

$$d_M = \begin{cases} 3 & \text{for} \quad 0^- \\ -1 & \text{for} \quad 1^- \end{cases}$$

$$d_{\mathcal{A}} = \begin{cases} 1 & \text{for} \quad H^*H \quad \text{coupling} \\ -1 & \text{for} \quad H^*H^* \quad \text{coupling} \end{cases} \tag{25}$$

and:

$$\varepsilon_1 = -m + G_H^2\left(\frac{\langle\,\overline{q}q\,\rangle}{4m} + f^2 + \frac{N_c m^2}{16\pi} + \frac{C_K}{16}(\frac{\langle\,\overline{q}q\,\rangle}{m} - f^2) + \frac{1}{128 m^2}(C_K + 8 - 3\pi)\langle\frac{\alpha_s}{\pi}G^2\rangle\right) , \tag{26}$$

$$g_1 = m - G_H^2\left(\frac{\langle\,\overline{q}q\,\rangle}{12m} + \frac{f^2}{6} + \frac{N_c m^2(3\pi+4)}{48\pi} - \frac{C_K}{16}(\frac{\langle\,\overline{q}q\,\rangle}{m} + 3f^2) + \frac{1}{64m^2}(C_K - 2\pi)\langle\frac{\alpha_s}{\pi}G^2\rangle\right) , \tag{27}$$

$$g_2 = \frac{(\pi+4)}{(\pi+2)}\frac{\mu_G^2}{6m} , \qquad \varepsilon_2 = -\frac{g_2}{2} . \tag{28}$$

4 The β term for $D^* \to D\gamma$

The chiral Lagrangian β-term has the form [4]:

$$\mathcal{L}_\beta = \frac{e\beta}{4} Tr[\overline{H}\, H\, \sigma\cdot F\, Q^\xi] , \tag{29}$$

where $Q^\xi = (\xi^\dagger Q\xi + \xi Q \xi^\dagger)/2$, $Q = diag(-2/3, \; -1/3, \; -1/3)$, and F is the electromagnetic filed tensor. The β term can be calculated in HLχQM, by considering the diagrams in Fig. 1. These diagrams will be calculated following [22], both for the gluon and the electromagnetic field. The first diagram without gluons is logarithmically divergent. Performing the calculation of the rest of the diagrams we used the algebraic program FORM [23]. We obtained the following expression:

$$\beta_{LO} = \frac{G_H^2}{2}\left\{-4iN_c I_2 + \frac{N_c}{4\pi} - \frac{1}{4m^4}\left(\frac{32+3\pi}{144}\right)\langle\frac{\alpha_s}{\pi}G^2\rangle\right\} . \tag{30}$$

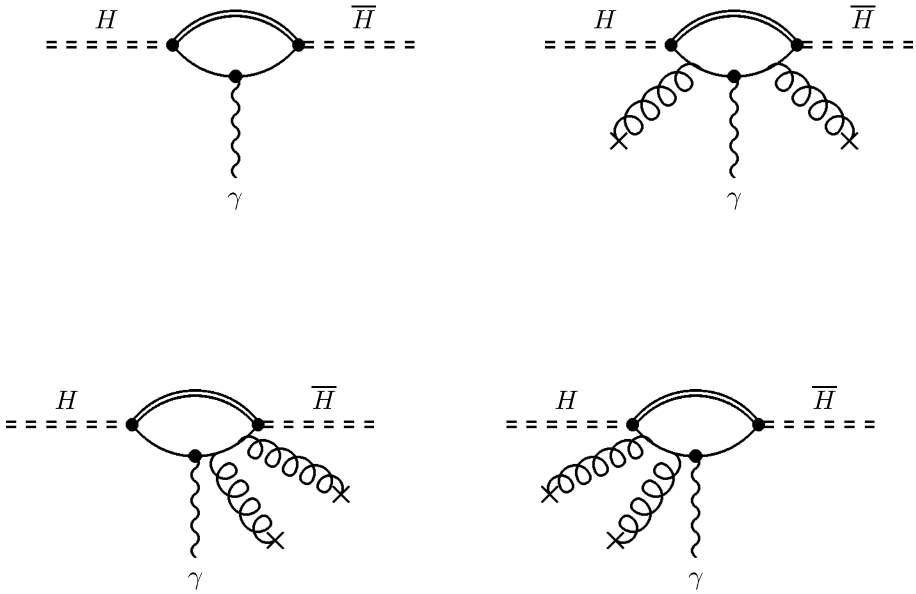

Fig. 1. Leading order diagrams contributing to β. The double and single lines represent heavy and light quarks, respectively. The double dashed lines represent heavy mesons, and the wavy lines represent emission of soft gluons ending in vacuum to make gluon condensates.

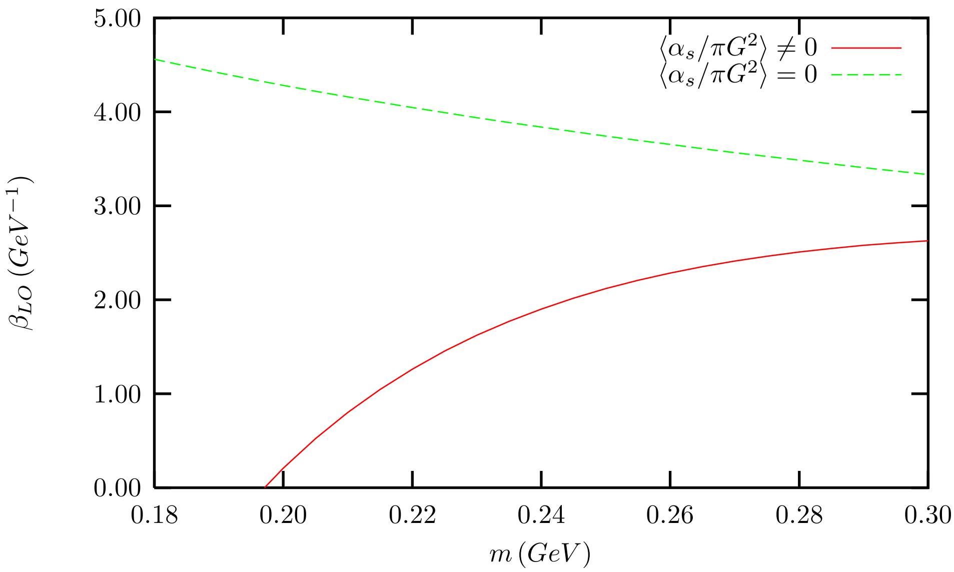

Fig. 2. β_{LO} as a function of the constituent light quark mass for parameters from [7](solid line). The dashed line shows the behaviour if the gluon condensate were omitted.

Using the relation (39) for the logarithmic divergent loop integral I_2, we obtain:

$$\beta_{LO} = \frac{G_H^2 f^2}{2m^2} \left\{ 1 + \frac{N_c m^2}{4\pi f^2} - \frac{1}{4 f^2 m^2} \left(\frac{56 + 3\pi}{144} \right) \langle \frac{\alpha_s}{\pi} G^2 \rangle \right\} . \tag{31}$$

As seen from Fig. 2, β depends strongly on the constituent light quark mass m, especially for m below 250 MeV. There is a partial cancellation between large terms in (31), and for values of m smaller than about 200 MeV, β_{LO} turns negative. Note also that if the gluon condensate is dropped in (31), β would be too big, as seen from Fig. 2.

Apriori one might hope that $1/m_Q$ corrections stabilizes the value for β for the range 190 to 250 MeV used in [7]. However, as seen in the following, this will not be the case. The $1/m_Q$ corrections to β can be found by calculating the diagrams in Figs. 3 and 4. We find that the kinetic term in (11) give a negative contribution to β, and the chromomagnetic term a positive contribution. However, the kinetic term dominates in absolute value, and in

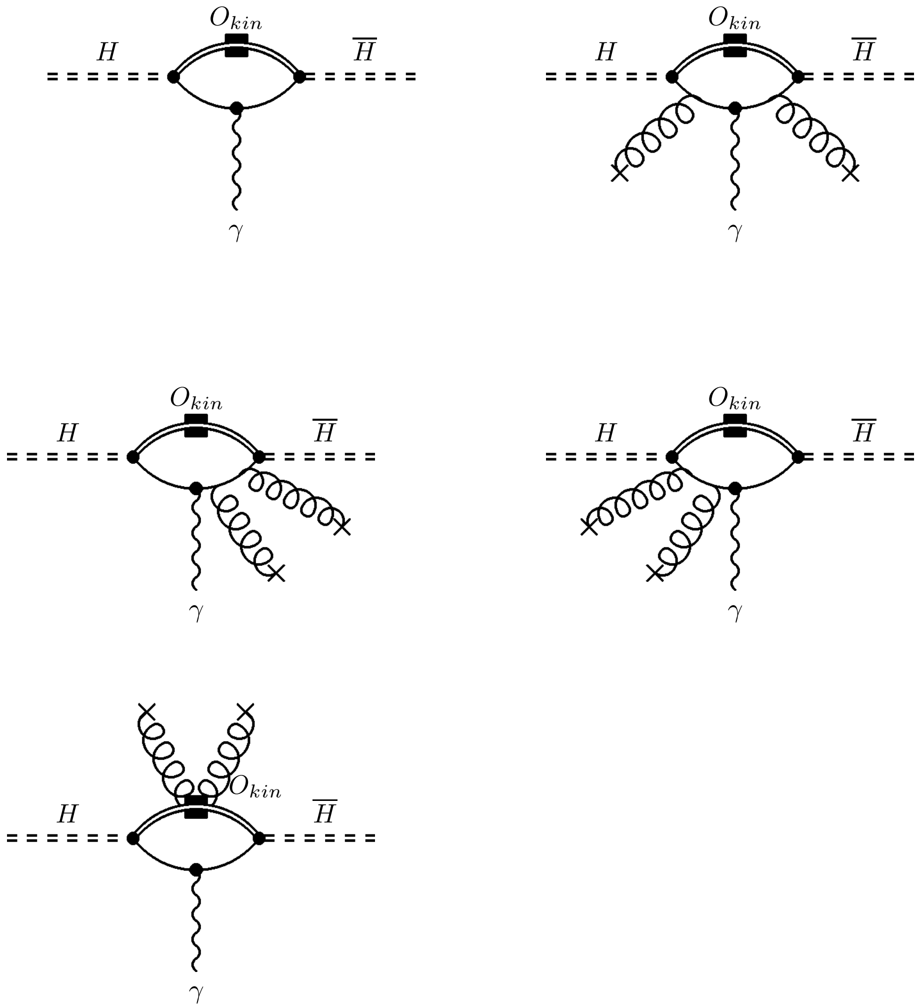

Fig. 3. $1/m_Q$ diagrams from the kinetic operator, contributing to β

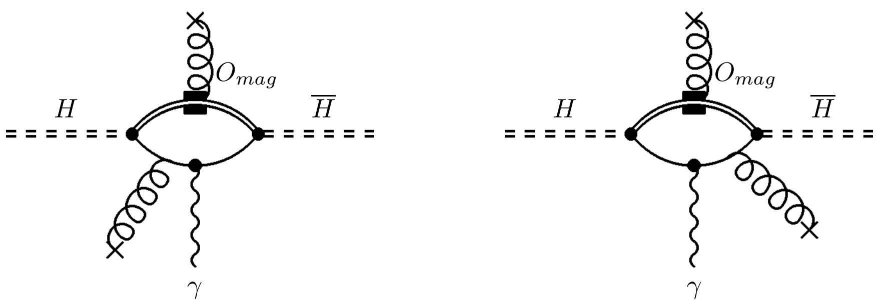

Fig. 4. $1/m_Q$ diagrams from the chromo magnetic operator, contributing to β

total, $1/m_Q$ corrections give a negative contribution to β for m below 250 MeV.

Combining the leading order and the $1/m_Q$ corrections we find:

$$\beta = \sqrt{Z_H Z_{H^*}}\beta_{LO} + \frac{\tau_\beta}{m_Q}\,,$$

where

$$\tau_\beta = -\frac{3}{4}(1-g_{\mathcal{A}}) - \frac{G_H^2}{4m}\left\{2f^2 - \frac{m^2 N_c}{2\pi^2} + \left[\frac{1}{12} + \frac{\pi}{192} - C_M\frac{(\pi+4)}{72}\right]\frac{1}{m^2}\langle\frac{\alpha_s}{\pi}G^2\rangle - C_K\left(f^2 - \frac{1}{36m^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right)\right\}\ . \tag{32}$$

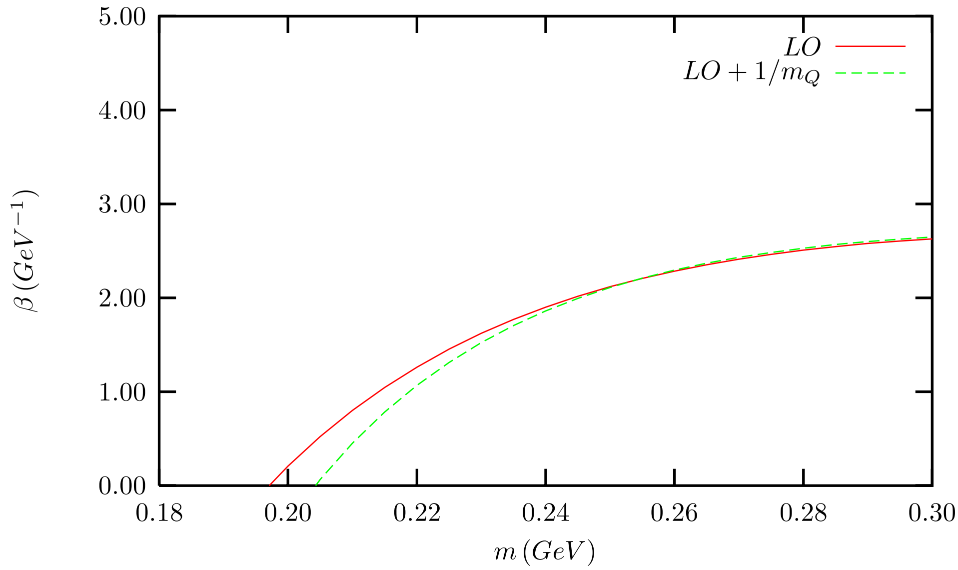

Fig. 5. β as a function of the constituent light quark mass for $g_{\mathcal{A}} = 0.59$

where τ_β contains the result of the diagrams in the Figs. 3 and 4. A plot of β is shown in Fig. 5. As we see, the result varies significantly with m. This is in contrast to other quantities studied in [7,11]. We observe that the quantities studied in [7,11] have contributions of zeroth order in m. In contrast, β is of order $1/m$, which means that β is expected to be more sensitive to variations of m. The $1/m_Q$ corrections are not sizeable, even if the charm quark is not very heavy. However, they pull in the wrong direction compared to the experimental value of order 2-3 GeV^{-1}. To obtain a value closer to the experimental value for β, we need a value for m higher than used in [7,11]. However, this will lead to values of f_B and f_D which are too small . This can be compensated by using a higher value of the quark condensate $\langle \overline{q}q \rangle$ than the one obtained in QCD sum rules, which was the one used in [7,11]. This might be acceptable because it is not clear that our model dependent quantities $\langle \overline{q}q \rangle$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ should be exactly those obtained in QCD sum rules. Taking a higher quark condensate and making a new fit we obtain the result given in the Tables 1 and 2. Note that the curve for β itself as a function of m does not change, but we are now allowed to go to higher values for m at the curve.

Chiral Lagrangian terms with extra current quark mass can be obtained by taking the derivative of the expression for β in (30) with respect to m, when keeping G_H and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ fixed. This gives:

$$\mathcal{L}_{FM} = \frac{e\widetilde{\alpha}}{8} Tr \left[\overline{H}\, H\, \sigma \cdot F \, \left(Q^\xi \, \widetilde{M}_q^V + \widetilde{M}_q^V Q^\xi \right) \right] \ , \qquad (33)$$

where

$$\widetilde{\alpha} = \frac{G_H^2}{4m} \left\{ -\frac{N_c}{\pi^2} + \left(\frac{32 + 3\pi}{72 m^4} \right) \langle \frac{\alpha_s}{\pi} G^2 \rangle \right\} . \qquad (34)$$

This term is unimportant for $D^* \to D\gamma$ because of the small current quark masses for the u, d quarks, but it gives a sizeable contribution to $D_s^* \to D_s \gamma$.

5 Conclusion

We have calculated the quantity β and have found that it depends significantly on the constituent light quark mass m. To obtain a value close to the experimental one has to pick values on the high side (m= 250 to 300 MeV, say) compared to the values in [7,11], where $m = 220 \pm 30$ MeV were used. We have redone the fits in [7,11] at the price of a higher value of the quark condensate $\langle \overline{q}q \rangle$ in the range $-(250 \text{ to } 270 \text{ MeV})^4$. The results are given in the Tables 1 and 2. For the case $D^* \to D\gamma$ we have found that (the bare) $\beta = (2.5 \pm 0.6)\ \mathrm{GeV}^{-1}$ to be compared with $\beta = (2.7 \pm 0.20)\ \mathrm{GeV}^{-1}$ extracted from experiment.

To conclude, we have calculated the quantity β relevant for $D^* \to D\gamma$, within a model exhibiting chiral- and heavy quark symmetry for a constituent light quark mass m in the range 190 to 300 MeV (where the range 250 to 300 MeV gives the best result for β). Because of the sensitivity of β with respect to m, and because we want to check the consistency of our model, we have in the appendix studied the limit $m \to 0$ for our model.

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Appendix A: The limit $m \to 0$

In this Appendix we will discuss the limit of restauration of chiral symmetry, i.e. the limit $m \to 0$. In order to do this, we have to consider the various constraints obtained when constructing the HLχQM [7].

To obtain (19) from the HLχQM one encounters divergent loop integrals, which will in general be quadratic-, linear- and logarithmic divergent. For the kinetic term we

Table 1. Predictions of HLχQM for different input parameters in the D-sector.

	$g_{\mathcal{A}}{}^{D^*D\pi} = 0.59 \pm 0.08$ [18] $\langle\,\overline{q}q\,\rangle = -(230-250)^3$ MeV $m = (190-250)$ MeV	$g_{\mathcal{A}}{}^{D^*D\pi} = 0.59 \pm 0.08$ [18] $\langle\,\overline{q}q\,\rangle = -(250-270)^3$ MeV $m = (250-300)$ MeV
G_D	(7.2 ± 0.6) GeV$^{-1/2}$	(6.4 ± 0.4) GeV$^{-1/2}$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4}$	(290 ± 20) MeV	(330 ± 20) MeV
g_1	(0.8 ± 0.1) GeV	(0.61 ± 0.07) GeV
g_2	(0.32 ± 0.04) GeV	(0.25 ± 0.02) GeV
ε_1	$-(0.8 \pm 0.3)$ GeV	$-(0.6 \pm 0.2)$ GeV
λ_1	0.8 ± 0.2	0.5 ± 0.1
μ_π^2	(0.34 ± 0.05) GeV^2	(0.29 ± 0.04) GeV^2
f_D	(220 ± 60) MeV	(220 ± 35) MeV
f_{D^*}	(260 ± 85) MeV	(235 ± 50) MeV
f_{D_s}	(245 ± 70) MeV	(260 ± 45) MeV
$f_{D_s^*}$	(280 ± 95) MeV	(270 ± 60) MeV
f_{D^*}/f_D	1.2 ± 0.1	1.12 ± 0.04
f_{D_s}/f_D	1.18 ± 0.06	1.26 ± 0.04
β	(0 ± 2) GeV^{-1}	(2.5 ± 0.6) GeV^{-1}

obtain the identification:

$$-iG_H^2 N_c \left(I_{3/2} + 2mI_2 + i\frac{(3\pi - 8)}{384 m^3 N_c} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right) = 1 \,, \tag{35}$$

and for the axial vector term

$$-iG_H^2 N_c \left(-\frac{1}{3} I_{3/2} + \frac{im}{12\pi} + 2mI_2 + i\frac{(3\pi - 8)}{384 m^3 N_c} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right) = g_{\mathcal{A}} \,. \tag{36}$$

Combining (35) and (36), the strong axial coupling $g_{\mathcal{A}}$ may be written:

$$g_{\mathcal{A}} = 1 - \delta g_{\mathcal{A}} \,, \quad \text{where} \quad \delta g_{\mathcal{A}} = -\frac{4}{3} i G_H^2 N_c \left(I_{3/2} - \frac{im}{16\pi} \right) \,. \tag{37}$$

Here the term 1 for $g_{\mathcal{A}}$ corresponds to the normalization (35) and the term $\delta g_{\mathcal{A}}$ is a dynamical deviation from this. It should be noted that also within other models [24,25], $g_{\mathcal{A}}$ may be written as $g_{\mathcal{A}} = 1 - \delta g_{\mathcal{A}}$, but the expression for $\delta g_{\mathcal{A}}$ is model dependent . We observe that the (formally) linear divergent integral $I_{3/2}$ is related to the strong axial coupling $g_{\mathcal{A}}$. Analogously, within the pure light quark sector (the χQM), it is well known that the quadratic and logarithmic divergent integrals are related to the quark condensate and f, respectively [8,13,14,15]:

$$\langle\,\overline{q}q\,\rangle = -4imN_c I_1 - \frac{1}{12m} \langle \frac{\alpha_s}{\pi} G^2 \rangle \,, \tag{38}$$

$$f^2 = -i4m^2 N_c I_2 + \frac{1}{24m^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle \,. \tag{39}$$

Eliminating $I_{3/2}$ from (35) and (37) and inserting the expression for I_2 obtained from (39) we find the following expression for G_H:

$$G_H^2 = \frac{m(1 + 3g_{\mathcal{A}})}{2f^2 + \frac{m^2 N_c}{4\pi} - \frac{\eta_1}{m^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle} \,, \quad \text{where} \quad \eta_1 \equiv \frac{\pi}{32} \,. \tag{40}$$

The divergent integrals I_1, I_2 and $I_{3/2}$ are (for $N = 1, 2$):

$$I_N \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)^N} \,, \qquad I_{3/2} \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{(v \cdot k)(k^2 - m^2)} \,, \tag{41}$$

where $I_{3/2}$ is proportional to the cut-off in primitive cut-off regularization:

$$I_{3/2} = i\frac{\Lambda}{16\pi} \left(1 + \mathcal{O}(\frac{m}{\Lambda}) \right) \,, \tag{42}$$

where the cut-off Λ (which we only have used in the qualitative considerations in this Appendix) is of the same order as the chiral symmetry breaking scale Λ_χ. In contrast, $I_{3/2}$ is finite and proportional to m in dimensional regularization. Within our model, the values for the regularized integrals I_1, I_2 and $I_{3/2}$ are determined by the numerical values of $\langle\,\overline{q}q\,\rangle$, f and $g_{\mathcal{A}}$, respectively.

Looking at the equations (38) and (39), one may worry [26] that $\langle\,\overline{q}q\,\rangle$ and f behaves like $1/m$ in the limit $m \to 0$ unless one assumes that $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ also go to zero in this limit. We should stress that the exact limit $m = 0$ cannot be taken because our loop integrals will then be meaningless. Still we may let m approach zero without going to this exact limit. In the pure light sector (at least when

Table 2. Predictions of HLχQM for different input parameters in the B-sector.

	$g_{\mathcal{A}}{}^{D^*D\pi} = 0.59 \pm 0.08$ [18] $\langle \overline{q}q \rangle = -(230-250)^3$ MeV $m = (190-250)$ MeV	$g_{\mathcal{A}}{}^{D^*D\pi} = 0.59 \pm 0.08$ [18] $\langle \overline{q}q \rangle = -(250-270)^3$ MeV $m = (250-300)$ MeV
G_B	(8.3 ± 0.7) GeV$^{-1/2}$	(7.2 ± 0.5) GeV$^{-1/2}$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4}$	(300 ± 25) MeV	(340 ± 20) MeV
g_1	(1.4 ± 0.3) GeV	(1.0 ± 0.2) GeV
g_2	(0.39 ± 0.05) GeV	(0.31 ± 0.03) GeV
ε_1	$-(0.9 \pm 0.4)$ GeV	$-(0.6 \pm 0.2)$ GeV
λ_1	1.1 ± 0.3	0.8 ± 0.1
μ_π^2	(0.42 ± 0.03) GeV^2	(0.39 ± 0.03) GeV^2
f_B	(190 ± 50) MeV	(185 ± 30) MeV
f_{B^*}	(200 ± 60) MeV	(190 ± 35) MeV
f_{B_s}	(210 ± 70) MeV	(215 ± 45) MeV
$f_{B_s^*}$	(220 ± 70) MeV	(215 ± 45) MeV
f_{B^*}/f_B	1.07 ± 0.04	1.02 ± 0.02
f_{B_s}/f_B	1.14 ± 0.07	1.22 ± 0.02
$\hat{B}_{B_d}$	1.51 ± 0.09	1.52 ± 0.07
$\hat{B}_{B_s}$	1.4 ± 0.1	1.4 ± 0.1
$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}$	1.08 ± 0.07	1.16 ± 0.04
β	$-(2 \pm 3)$ GeV^{-1}	(1.2 ± 0.8) GeV^{-1}

vector mesons are not included) there are no restrictions on how $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ might go to zero. In the heavy light sector we have in addition to (38) and (39) also the relations (35), (37), and (40) which put restrictions on the behavior of the gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ for small masses. As $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ has dimension mass to the fourth power, we find that $\langle \overline{q}q \rangle$ and f^2 may go to zero if $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ goes to zero as m^4 or $m^3 \Lambda$ (eventually combined with $\ln(m/\Lambda)$). However, the behavior $m^3 \Lambda$ is inconsistent with the additional equations (20) and (21). Still, from all equations (35)- (40) and (20), (21), we find the possible solution

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = \hat{c}\, N_c m^4 K(m)\,, \qquad \text{where} \quad K(m) \equiv (-4iI_2 + \frac{1}{8\pi})\,, \tag{43}$$

and $\hat{c}$ is some constant. Then we must have the following behavior for G_H^2, $g_{\mathcal{A}}$ and μ_G^2 when m approaches zero:

$$G_H^2 \sim \frac{1}{N_c \Lambda}\,, \qquad (1 + 3g_{\mathcal{A}}) \sim \frac{m}{\Lambda} K(m)\,, \qquad \mu_G^2 \sim \frac{m^3}{\Lambda} K(m)\,, \tag{44}$$

with some restrictions on the proportionality factors. Here, the regularized I_2 is such that for small m, $K(m) = (c_1 + c_2 \ln m/\Lambda)$, c_1 and c_2 being constants. The behavior of G_H^2 is in agreement with Nambu-Jona-Lasinio models [9]. Note that in our model, $\delta g_{\mathcal{A}} \to 4/3$ (corresponding to $g_{\mathcal{A}} \to -1/3$) for $m \to 0$, in contrast to $\delta g_{\mathcal{A}} \to 2/3$ in [24] for a free Dirac particle with $m = 0$. Note that in [7] we gave the variation of the gluon condensate with m for a fixed value of μ_G^2. For the considerations in this Appendix, we have to let μ_G^2 go to zero with m in order to be consistent. When $m \to 0$, we also find that $\beta \to 1/\Lambda$, provided that the coefficient $\hat{c}$ in (43) is fixed to a specific value (which is $\hat{c} = 576/(3\pi + 32) \simeq (1.93)^4$).

Before closing this section, we will shortly comment that in the limit where only the leading logarithmic integral I_2 is kept [7], when $g_{\mathcal{A}} \to 1$ [1] and $\rho \to 1$, we obtain from equation (31) the non relativistic quark model result [4,1]:

$$\beta \to \beta_{\mathrm{NR}} = \frac{1}{m}\,. \tag{45}$$

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